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# A Comparative Analysis of Credit Pricing Models
# Merton, CreditGrades™ and Beyond

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Financial institutions provide financing to their corporate clients in the form of loans, bonds, credit lines, and structured products such as asset backed securities. This lending activity leads to credit risk that needs to be quantified and managed. The recent development of the credit default swap market has enabled financial institutions to better manage their credit risk exposure. However, the liquidity in the credit default swap market is limited to a few hundred names, while financial institutions have an exposure to thousands of firms. Therefore, financial institutions still need other meaningful ways to assess the credit quality of firms lacking presence in the credit default swap market.

In 2002, Deutsche Bank, Goldman Sachs, JPMorgan and the RiskMetrics Group developed a completely transparent market based model – CreditGrades™ – to match modeled spreads with the observed spreads.

In this analysis, we compare the CreditGrades model with some simple regression-based models and the Merton-like probabilistic models to assess whether the probabilistic models such as CreditGrades provide any added value over regression-based models; whether the results achieved by CreditGrades could be achieved by a Merton-like probabilistic model; and the extent to which an enhanced model of this nature can be built that would have better performance than CreditGrades.

Based on our review, we conclude that the random default barrier of the CreditGrades model may have a more significant effect for firms with higher credit spreads, and perhaps short term securities for which market data is not as readily available are also more significantly affected by the CreditGrades variable default barrier.

We observe that the probabilistic models perform better in matching the credit spread movement than the regression-based models, but have no significant advantage in pricing of the credit spreads for individual issuers.

We develop a modified Merton model that is less prone than the standard Merton model to undervalue credit spreads and produces credit spreads closer to the observed than the CreditGrades model more than 60% of the time.

We find suggestive evidence that in a dynamic marketplace, CreditGrades predictive abilities may at times lag behind the market, suggesting the need for the development of tools to make better use of implied equity and asset volatilities in lieu of or in addition to historical equity volatility.

Finally, we identify an approach to resolve difficulties present in determining the level of debt and default barriers that enables us to quantify these variables, and, thereby to better predict credit spreads.

Financial institutions provide financing to their corporate clients in the form of loans, bonds, credit lines, and structured products such as asset backed securities. This lending activity leads to credit risk that needs to be quantified and managed. The recent development of the credit default swap market has enabled financial institutions to better manage their credit risk exposure. However, the liquidity in the credit default swap market is limited to a few hundred names, while financial institutions have an exposure to thousands of firms. Therefore, financial institutions still need other meaningful ways to assess the credit quality of firms lacking presence in the credit default swap market.

To do a quantitative market valuation of the credit exposure, Robert Merton proposed in 1974 to view a company's equity as a call option on its assets, thereby linking credit and equity valuation. This initiated the use and further development of structural models. To more closely match modeled spreads with observed spreads, Deutsche Bank, Goldman Sachs, JPMorgan and the RiskMetrics Group, developed the CreditGrades model in 2002 as a completely transparent market based model that resolves certain empirical inconsistencies with the standard Merton model. The CreditGrades model is now used to rank credits and is available on the web at www.creditgrades.com.

In this analysis, we examine CreditGrades model performance relative to that of the probabilistic and regression-based models under different market conditions and attempt to find a structural, transparent market-based model that could have better performance than the CreditGrades model.

This paper is divided into three parts. In the first part, we present the theoretical background behind the models used for analysis. In the second part of the paper, we provide empirical results. Finally, based on our analysis, we conclude with some reasonably intuitive approaches to consider in further research.

In our model comparison analysis, we considered three types of models: Merton model and its variants, CreditGrades model and its variants, and the least-square regression-based models. In this section, we present the detailed description of each model. The implementation issues and approach for some of these models are presented in Appendix A.

**Merton Model**

When a corporation is unable to service its outstanding liabilities, the company defaults on its debt and begins the process of restructuring or, in extreme cases, liquidation. In such case, the equity holders are the last in line of stakeholders to make a claim on the company's assets.

Let $A_T$ denote the value of a company's assets at time T. Also, let $D_T$ denote the face value of all outstanding liabilities at time T. Then presuming that a company ceases its operations at time T and has no other outstanding debt due at a later date, the payoff to the equity holders, $E_T$, can be written as:

$$E_T = \max(A_T - D_T, 0) \qquad 1.1$$

Of course, the above expression constitutes the payoff of the call option on $A_T$ struck at $D_T$. Therefore, presuming that for $t \in [0, T]$:

(i) $A_t$ follows a lognormal diffusion process $dA_t = \mu A_t dt + \sigma A_t dW_t$;
(ii) there is continuous trading of assets;
(iii) the Modigliani-Miller theorem holds in the sense that the value of the firm is invariant to its capital structure;
(iv) the term structure of interest rates is flat;
(v) the volatility, $\sigma_A$, is constant;
(vi) all outstanding liabilities consist of a non-callable zero coupon bond with face value of $D_T$ maturing at time T;
(vii) there is dilution and bankruptcy protection;

the equity value, $E_t$, should satisfy the Black-Scholes option pricing formula for the European call

$$E_t = A_t N(d_1) - e^{-r(T-t)} D_T N(d_2) \qquad 1.2$$

where $N(\cdot)$ is the cumulative normal distribution function and

$$d_1 = \frac{\log(A_t/D_T) + (r + ½\sigma_A^2)(T - t)}{\sigma_A (T - t)^{½}}$$

$$d_2 = d_1 - \sigma_A (T - t)^{½}$$

The yield to maturity, y, of a bond is the solution to

$$D_t = e^{-y(T-t)}D_T = e^{-(s+r)(T-t)}D_T \qquad 1.3$$

where s is the spread above the risk-free rate r and $D_t$ denotes the market value of liabilities at time t.

Therefore, using the accounting identity, assets = liabilities + equity, the equity value, $E_t$, should be

$$E_t = A_t - D_t = A_t - e^{-(s+r)(T-t)}D_T \qquad 1.4$$

Combining equations 1.2 and 1.4 leads to

$$A_t - e^{-(s+r)(T-t)}D_T = A_t N(d_1) - e^{-r(T-t)}D_T N(d_2)$$

$$e^{-s(T-t)} = A_t e^{r(T-t)}N(-d_1)/D_T + N(d_2)$$

$$s = -\frac{\log\{A_t e^{r(T-t)}N(-d_1)/D_T + N(d_2)\}}{(T-t)} \qquad 1.5$$

The values of $A_t$ and $\sigma_A$ in equations 1.2 and 1.5 are unknown. However, it is straightforward to determine them by solving equation 1.2 and the following equation 1.6, simultaneously.

$$E_t\sigma_E = A_t N(d_1)\sigma_A \qquad 1.6$$

The above constitutes the standard Merton model first proposed by Robert Merton in 1974. The risk-neutral probability of default in this model is the probability that an investor will not exercise the call option to buy the assets of the firm at time T and is given by $N(-d_2)$. Moreover, the recovery rate, R, is $N(-d_1)/N(-d_2)$ and the loss given default is

$$L_D = 1 - RA_t e^{r(T-t)}/D_T \qquad 1.7$$

**Merton Model Variants**

*Variant 1 – L50:*

In the Merton model, the loss given default is given by equation 1.7. Since a real-world default is often a protracted negotiation, this formulaically derived loss on default may not be reasonable. Perhaps the loss on default does in fact reflect the relatively large amount of legal and other bankruptcy costs associated with the default process and the protracted negotiations involved therein. Accordingly, in this variant of the Merton model, the loss on default is set equal to 50% of the face value of the debt.

*Variant 2 – Jump:*

Empirically, credit spreads generated by the Merton model for short maturities and investment grade firms are much lower than those observed in the market, suggesting that some type of jump process may in fact also be in play. This is consistent with Gatheral's observation that equity option prices shortly before expiration are often priced for four or five standard deviation moves, which can only be explained by the inclusion of jumps in the model.

The model called Jump thus developed is a variant of the basic Merton model, in which a downward jump in the value of the firm occurs in 40% of the scenarios with this downward jump equal to 20% of the debt of the firm. The rationale for this form of jump is that firms with greater amounts of debt may be more likely to lose value due to unexpected problems. Other forms of jump such as letting the jump be equal to 20% of the equity of the firm were also considered but these did not give as strong results. The rationale for such a form of jump would have been that a firm with a strong stock price is more likely to experience a major surprise than is a firm with a somewhat weaker stock price. Use of a jump in the asset price may be a suitable alternative to the CreditGrades approach of using a jump in the default barrier.

*Variant 3 – SVWavgV:*

In this model, there are two variations on the basic Merton model. First, the volatility of the firm's value is adjusted upwards for firms with low Merton volatility and downwards for firms with high Merton volatility. The rationale for this adjustment is that it was felt that the markets may anticipate a certain level of unexpected surprises and, therefore, assume a higher volatility than that actually being experienced for firms with low volatility. On the other hand, for firms with high volatility, it was felt that the markets may consider the high current volatility to be partially related to excess concern at the time that bad news emerges and the market therefore anticipates that some of this excess volatility may be mitigated once the market absorbs the complete scope of the transpiring events. This theory tends to be supported by a parallel with the equity markets observed by Hull that the volatility term structure tends to be downward sloping when volatility is high and upward sloping when it is low. In our implementation in this paper, the firm's volatility for modeling purposes is equal to the average of the Merton computed volatility and .4.

The second variation on the basic Merton model is that Stochastic Volatility is used. As Gatheral notes regarding equities, modeling volatility as a random variable seems suitable for equities because equities exhibit "volatility clustering" in which large moves follow large moves and small moves follow small moves. In addition, the distribution of stock price returns is highly peaked and fat-tailed relative to the Normal distribution, which is characteristic of mixtures of distributions with different variances. Likewise, in fixed income, it is well known that bond defaults tend to occur in clusters and peak and trough at different points in the economic cycle, again with a highly peaked and fat-tailed distribution. Gatheral also notes that there is a simple economic argument which justifies the mean reversion of volatility, namely that the likelihood of volatility being between 1% and 100% in 100 years time would be very low if volatility were not mean reverting. Since we believe that volatility will be in this range, it is to be expected that volatility is a mean reverting random variable. A similar argument could be made for the volatility of defaults as we certainly expect that this volatility would remain bounded between 1% and 100% over the next 100 years. It is then straightforward to extend these arguments to the asset value of the firm.

In this implementation called SVWagvV, the volatility of variance used is .4 and the assumed correlation between the random variables for stochastic variance and for stochastic return on firm value was .6. In order to reflect the full effects of the stochastic volatility, each scenario is given a different stochastically generated volatility which is used for the entire duration of the scenario.

Although we noted various parallels for stochastic volatility with the equity markets, it is also worthwhile to note that various authors such as Lewis have observed that implied equity option volatility ultimately flattens to a limiting asymptotic value as a function of time and that this limiting asymptotic value is independent of the moneyness and the initial volatility. Since the credit spreads analyzed in this paper are for terms of five years, it would be worthwhile to analyze whether a similar result holds for credit spreads.

*Variant 4 – Hull:*

While the standard Merton model provides the relationship between equity prices and credit spreads, the more recent research has been involved looking for the relationship of the credit default probabilities and the volatility skews or surface of equity options.

Since all stock issuers have some probability of default, the option prices should reflect some of the default risk. The observed equity volatility skew has been explained by the fact that most of the equity investors have long position and they seek the downside protection. Moreover, the issuing corporations are the supply source and they do not seek protection against price increases. Therefore, there is far greater demand for the out-the-money puts than the demand for the out-the-money calls. This leads to the volatility skew. However, the steepness of the volatility skew should reflect the issuer's probability of default since the greater the likelihood of default means the greater the likelihood of the drop in the price of the security, thereby, the higher the price in term of volatility the long equity investor would pay for the downside protection.

The model called Hull is a variant of the Merton model proposed by Hull, et. al. designed to capture the credit default probability from the volatility skew observed in the equity market. The model relies only on the market data and, therefore, might be expected to provide better results than standard Merton model, which may be skewed by the over/under estimating of the results on the balance sheets of the issuers. In implementing this model, implied volatilities of two two-month options – one near at-the-money, the other out-of-the-money – are used to compute equation 1.5 inputs, $\sigma_A$ and the leverage ratio, $D_t/A_t$.

**CreditGrades Model**

The CreditGrades model is built on the framework of the Black-Cox model, which relaxes some of the assumptions present in the standard Merton model to enable default to occur prior to time T if the value of the company assets hits a predetermined default barrier.

The CreditGrades model uses the uncertainty in the default barrier to address artificially low short-term spreads present in other structural models since the asset value starting above the barrier cannot reach the barrier in the next instance by diffusion alone.

Assuming that:

(i) $A_t$ follows a lognormal diffusion process $dA_t = \mu A_t dt + \sigma A_t dW_t$. Moreover, $A_t$ is a martingale;
(ii) default occurs the first time $A_t$ crosses the default barrier defined as the recovery value that the debt holder receives and given by the product of the firm's debt-per-share, D, and the recovery rate, L;
(iii) the firm's debt-per-share, D, is defined as

$$D = \frac{\text{FINANCIAL DEBT} - \text{MINORITY INTEREST}}{\text{NUMBER OF SHARES}} \qquad 1.8$$

FINANCIAL DEBT = (ST DEBT + LT DEBT) + ½(ST LIABILITIES + LT LIABILITIES)

NUMBER OF SHARES = COMMON SHARES + PREFERRED EQUITY/STOCK PRICE

(iv) the recovery rate, L, is stochastic and follows a lognormal distribution with $E[L] = L_m$, $Var[\log(L)] = \lambda^2$, and $LD = L_m D\exp(\lambda Z - ½\lambda^2)$ where Z is normally distributed random variable.

For an initial value $A_0$, default does not occur as long as

$$A_0\exp\{\sigma W_t - ½\sigma^2 t\} > L_m D\exp(\lambda Z - ½\lambda^2) \qquad 1.9$$

This leads to the survival probability at time t, $P_t$, given by

$$P_t = N(-½Q_t + \log(d)/Q_t) - dN(-½A_t - \log(d)/Q_t) \qquad 1.10$$

where

$$d = A_0\exp(\lambda^2)/ L_m D$$

$$Q_t^2 = \sigma^2 t + \lambda^2$$

To complete the model derivation, assume that $A_t$ can be expressed as

$$A_t = E_t + LD \qquad 1.11$$

Define a distance to default measure, $\eta$, as the number of annualized standard deviations separating the firm's current equity value from the default threshold expressed as

$$\eta = \frac{A_t \log(A_t/LD)}{\sigma_E E_t} = \qquad 1.12$$

$$= \frac{(E_t + LD)}{\sigma_E E_t} \log\left(\frac{E_t + LD}{LD}\right)$$

Then at time t = 0, the initial value $A_0$ is

$$A_0 = E_0 + L_m D \qquad 1.13$$

This leads to an expression linking asset volatility, $\sigma$, to observable equity volatility, $\sigma_E$

$$\sigma = \sigma_E E_t/(E_t + L_m D) \qquad 1.14$$

Now we can calculate the survival probability, $P_t$, using observable market parameters.

For constant risk-free rate, r, and the survival probability given by equation 1.10, the spread, s, at time t can be expressed as

$$s_t = \frac{r(1 - R)\{1 - P_0 + e^{r\xi}(G(t + \xi) - G(t))\}}{P_0 - e^{rt}P_t - e^{r\xi}(G(t + \xi) - G(t))} \qquad 1.15$$

where R is the asset specific recovery rate, $\xi = \lambda^2/\sigma^2$, and function G is given by

$$G(t) = d^{z + ½}N\left(\frac{-\log(d) - z\sigma^2 t}{\sigma t^{½}}\right) + d^{-z + ½}N\left(\frac{-\log(d) + z\sigma^2 t}{\sigma t^{½}}\right)$$

with $z = (¼ + 2r/\sigma^2)^{½}$

**CreditGrades Model Variants**

As an alternative to the CreditGrades model, we considered redefining the debt-per-share measure in the assumption (iii).

*Variant 1 – CG2:*

Following the definition provided in the Moody's KMV model, we created a model CG2 by redefining financial debt in equation 1.18 as

FINANCIAL DEBT = (ST DEBT + ST LIABILITIES) + ½(LT DEBT + LT LIABILITIES)

*Variant 2 – CG3:*

In this alternative called CG3, we changed the definition of the financial debt to the pure book value of the outstanding liabilities. Thus, equation 1.18 became

FINANCIAL DEBT = ST DEBT + ST LIABILITIES + LT DEBT + LT LIABILITIES

## The Least-Square Regression Models

*Credit Rating Score Regression*

Many corporate debt issuers are assigned a publicly available credit rating by credit rating agencies (Fitch, Moody's, and Standard & Poor's). Some market participants place a lot of trust in the analysis performed by credit rating agencies; others use them as an initial classification of the riskiness of the obligator. Hence, the credit ratings have a significant impact on the price of the corporate debt of the rated issuers.

To compare the accuracy of the structural models detailed before to an alternative simple credit rating based model, we developed the following least-square regression-based model.

Assigning each credit rating a so-called credit rating score (Table 1), we regressed the credit spreads observed in the market versus the credit rating score to determine the most optimal-fitting model of each analyzed period. It turned out that the exponential regression model of the form $y = \beta e^{\alpha x} + \varepsilon$ provided the best fit. To arrive at the final model, we computed the average regression parameters, $\alpha$ and $\beta$.

| S&P Credit Rating | Credit Rating Score | S&P Credit Rating | Credit Rating Score | S&P Credit Rating | Credit Rating Score |
|---|---|---|---|---|---|
| AAA | 1 | BBB | 9 | CCC+ | 17 |
| AA+ | 2 | BBB- | 10 | CCC | 18 |
| AA | 3 | BB+ | 11 | CCC- | 19 |
| AA- | 4 | BB | 12 | CC+ | 20 |
| A+ | 5 | BB- | 13 | CC | 21 |
| A | 6 | B+ | 14 | CC- | 22 |
| A- | 7 | B | 15 | C+ | 23 |
| BBB+ | 8 | B- | 16 | C | 24 |

Table 1 – Credit Rating Scores

*Distance to Default Regression*

The weakness of any credit rating based model is that this type of model does not reflect the movement in the general level of the credit spreads, as credit rating agencies do not change their rating to correspond to the changes in the business cycle. Therefore, unless the credit rating agency adjusts the credit rating, according to the credit rating based model, a firm would maintain a constant credit spread.

A more advanced model would incorporate a measure that can reflect the general movement in the credit spreads. Moody's KMV combines asset value, $A_t$, business risk, $\sigma_A$, and leverage into a single measure of the default risk called distance to default, which compares the market net worth to the size of a one standard deviation move in the asset value. One can view the distance to default measure as how far the company asset value is from some default point measured in the number of standard deviations.

Intuitively, the distance to default measure should move inversely to the credit spread, i.e. when credit spreads widen, the distance to default should decrease and conversely, when credit spreads narrow, the distance to default should increase.

Moody's KMV relies upon the distance to default measure and an extensive historic defaults database to compute an expected frequency of default measure which it uses to compute default probabilities and the associated credit spreads. Lacking Moody's KMV database, we developed two simpler alternative models based on the least-square regression of the credit spreads versus the following two definitions of the distance to default measure.

*Variant 1 – Distance to Default (DD) KMV:*

For this model called Distance to Default KMV, we used the distance to default definition provided by the Moody's KMV

$$DD_t = \frac{(A_t - DP)}{A_t \sigma_A} \qquad 1.16$$

where asset value, $A_t$, and asset volatility, $\sigma_A$, are computed using Merton model and default point, DP, is defined as a sum of all short term liabilities and half of long term liabilities.

The regression of the form $y = \beta + \alpha/x + \varepsilon$ provided the best fit.

*Variant 2 – Distance to Default (DD) CG:*

For this model called Distance to Default CG, we used an alternative definition of the distance to default provided in the CreditGrades model (equation 1.12).

The regression of the form $y = \beta e^{-\alpha x} + \varepsilon$ provided the best fit.

In this section, we present results of our analysis. We compare performance of the models detailed in the previous section and draw conclusions based on the empirical analysis.

**Study Dates**

Four dates were selected for analysis to give a sense as to how the various models operate in different spread environments. The dates selected for analysis were the following:

- January 5th, 2001 — This date corresponds to a period after the stock market peak in early 2000, but before the situations with companies such as Enron and WorldCom were well known. The average spread of the securities studied at this time was 118 basis points.
- October 15th, 2002 — This date corresponds to a period when credit spreads peaked. The average spread of the securities studied at this time was 168 basis points.
- September 30th, 2003 — Spreads declined after October 2002. This date corresponds to a period when spreads were relatively tight and in a downward trend. The average spread of the securities studied at this time was 64 basis points.
- September 29th, 2004 — A recent date when credit spreads were still relatively tight but starting to pick up again. The average spread of the securities studied at this time was 50 basis points.

The exhibit below indicates the average spread movement of the securities studied during the analysis period:

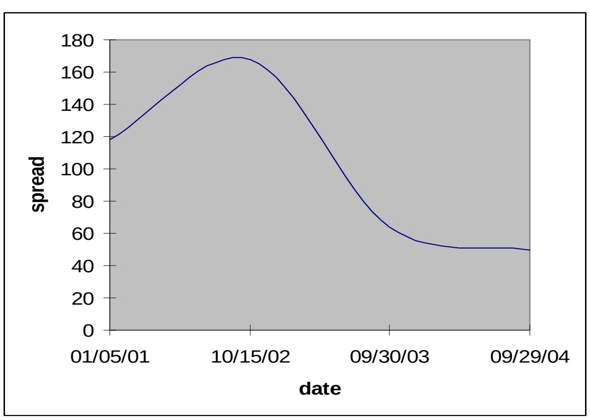


Exhibit 1 – Average Spread Movement

Note that all the models analyzed except for the Hull model were analyzed on each of these dates. The Hull model was analyzed on all dates except for January 5th, 2001 due to volatility skew data not being available for this date.

**Data**

Our empirical tests were based on the credit default swap data and equity price data provided by the Riskmetrics Group and the balance sheet data and the implied volatility data obtained from Bloomberg. The Riskmetrics Group also provided us with additional data – global recovery and asset recovery rates along with recovery rate volatility – to implement the CreditGrades model. JPMorgan Chase provided us with five-year credit default swap spreads on a list of liquid North American names. This data for names for which we have credit default swap spreads was used as a guide for names for which such spread data may not exist.

The credit default swap data was used to obtain the market value of the credit spreads to serve as a reference in our comparisons. We used only five-year quotes for US corporate names that have liquidity in the credit default market. No sovereign or quasi-sovereign names were used in the analysis. With the help of JPMorgan Chase, we used only names for which good individually marked, non-generic, public data exists. Furthermore, the US corporate names were split between the financial services companies ("financials") and other companies ("corporates").

The equity price data consisted of the closing equity prices for the dates analyzed and the historic volatility, which was based on the 1000 daily returns observed prior to each study date.

We used the reported nearest to the analysis date quarterly balance sheet data provided by Bloomberg to obtain relevant liability data.

The Bloomberg data was also used to obtain implied equity volatility data since it has an archive of the implied volatility for the US equities. Following the implementation presented in the Hull, we used the implied volatility for the two-month 50-delta put and the 25-delta put in the analysis.

Combined data yielded only 42[1] corporate names to use for meaningful analysis across all study dates. Within each study date, all corporate names were used to analyze results for all models. For the financials, the balance sheet data contained short-term liabilities such as repurchasing agreements used in the daily operations. This made comparison between some of the models difficult. Therefore, for each study date, we present the financials separately and only for some of the Merton models and the CreditGrades results provided by the Riskmetrics Group. To see if

[1] Upon review of the results obtained, we observed that S&P downgraded one company, Crown Cork & Seal (CCK), to a CC credit rating during the study period, and felt, therefore, that their results may have been having an unduly large influence on our overall findings. This led us to analyze in some instances only the 41 corporate names over the study period.

any model performs better for more risky issuers, we also compared results for the third most risky issuers measured by the observed credit spreads versus the results for the remaining issuers. Except where otherwise noted, we used the RiskMetrics determination of debt levels in the analysis.

**Statistical Measures**

We relied on the following four basic measures to compare each model:

(i) *Average Deviation* was obtained by taking the difference between the spread produced by the model and the actual credit spread for the name. The average of these differences for all names was then computed.

(ii) *Average Percentage Deviation* was obtained by taking the percentage difference between the spread produced by the model and the actual credit spread for the name. The average of these percentage differences for all names was then computed.

(iii) *Average Absolute Deviation* was obtained by taking the absolute value of the difference obtained in (i) above for each name. The average of these absolute values of difference for all names was then computed.

(iv) *Average Absolute Percentage Deviation* was obtained by taking the absolute value of the percentage difference obtained in (ii) above for each name. The average of these absolute values of percentage difference for all names was then computed.

Average deviations can guide us to how well the model matches observed data for the entire market; while absolute deviations give insight into how well the model matches observed data for specific debt issuers.

We also considered the rank correlation statistics presented in the CreditGrades Technical Document, but found in our preliminary work that the differentials between models were not quite as large on this basis as we might have anticipated and therefore chose to focus our efforts instead on the measures noted above.

## Results and Analysis

In the first part of this section, we present results across all study dates. We first use key models – credit ratings score regression, standard and L50 Merton models, and the CreditGrades model – to draw conclusions about their performances relative to the market and develop intuitive improvements to these models. We then use additional models to examine how the models performed relative to each other. In the second part of this section, we present results and draw conclusions within each study date.

### *Analysis across Study Period*

*Credit Spread Movement*

The following exhibit illustrates the change in average spreads between study dates for the key models:

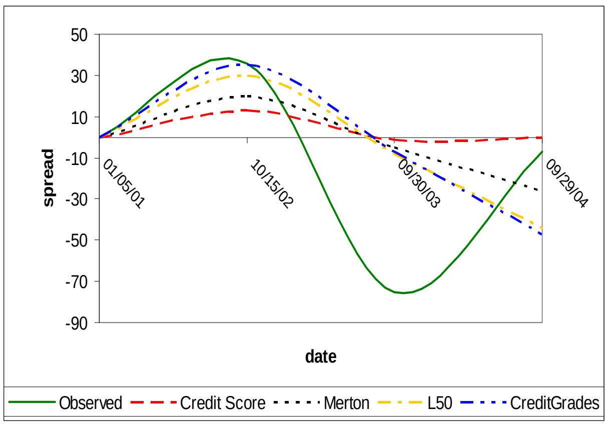


Exhibit 2 – Average Change in Spreads

The significant variation in market spreads on the different study dates is not reflected in the credit ratings score model. The initial up-tick in the average spreads generated by the credit ratings score model is followed by the straight line. This demonstrates the weakness of this type of models and suggests that the probabilistic models may add quantitative value over that provided by the credit rating based models.

Although the standard Merton, L50, and the CreditGrades models track the initial increase in market spreads between January 5th, 2001 and October 15th, 2002 study dates, the subsequent decline in market spreads between October 15th, 2002 and September 30th, 2003 study dates is not followed by any of the models. However, while market spreads were relatively flat between September 30th, 2003 and September 29th, 2004 study dates, both L50 and CreditGrades appear to make up for their previous shortfall as they have relatively large declines in average spreads. This seems to suggest that these probabilistic models track the market with a lag. Perhaps use of an implied volatility to supplement or to replace the historical volatility would help these models to better reflect current market conditions in real time.

Recognizing both the apparent lag just described and the fact that real-world default is often a protracted negotiation, and therefore estimations of loss upon default need to reflect the relatively large amount of legal and other bankruptcy costs associated with the default process in addition to the determination of expected payoff using Black-Scholes analysis, we fit the Merton model using current stock price and estimated loss upon default. The estimated loss upon default was selected to assure that the average deviation from actual market spreads would be close to zero. Although not implemented here, a straightforward extension of this model would be to vary the estimated loss upon default based upon credit rating.

The equity and asset volatilities developed in this implementation were therefore "implied volatilities" and not "historical volatilities". We found that these implied volatilities were about 35% above the actual historical volatilities on all study dates except for September 30th, 2003, when the implied volatilities exceeded the historical ones by about 15%. Likewise, the loss upon default exceeded that predicted by the standard Merton model by about 45% on all study dates except for September 30, 2003, when the excess was about 15%. This suggests that the declines in implied volatility and anticipated loss on default between October 15, 2002 and September 30, 2003 study dates were significantly greater than the corresponding relatively small declines in historical equity volatilities and standard implementation Merton computed losses upon default. As a result, model implementations that relied on historical equity volatility did not pick up the decline in spreads between these two dates.

To test this model, we can compare the implied equity volatilities thus developed with equity option implied equity volatilities. In addition, we can analyze the relationship between the resultant implied losses upon default and those observed statistically in the market.

A Merton implementation described above that relies on current stock price and estimated loss upon default chosen to assure zero average deviations could be used to predict credit spreads by determining the relationship between implied and historical volatility for names for which market spreads are available and then using this information to adjust historical volatility in an otherwise standard Merton model implementation for credit spread predictions for firms for which market spreads are not available. This approach might be somewhat more reliable than simply using the computed average implied loss upon default to fit the model because our proposed approach retains more firm specific information.

Another way in which to incorporate the observation that implied volatilities exceed historical volatilities is to work with the standard Merton model implementation and to add in jumps to ruin. The probability of jump to ruin can be determined for each name by determining the percentage adjustment to the asset value needed to match the Merton computed spread with the observed spread. Average probabilities of jump to ruin computed in this manner for each credit rating are shown in the following table:

| S&P Credit Rating | January 5th, 2001 | October 15th, 2002 | September 30th, 2003 | September 29th, 2004 |
|---|---|---|---|---|
| AA | 0.0010 | 0.0003 | 0.0001 | 0.0002 |
| A | 0.0016 | 0.0017 | 0.0001 | 0.0006 |
| BBB | 0.0032 | 0.0041 | 0.0007 | 0.0015 |

Table 2 – Annualized Implied Probability of Jump to Ruin

The changes in implied probabilities between January 5, 2001 and October 15, 2002 study dates appear to reflect both the apparent tightening in criteria for determination of high investment grade credit ratings and the apparent worsening of credit conditions for the market as a whole. The extremely low implied probabilities on the September 30, 2003 study date appear to reflect that historical volatilities trail the market in terms of the information they contain and the implied probabilities on the September 29, 2004 study date appear to be more realistic for a low spread environment.

Although these implied probabilities of jump to ruin are computed using publicly available information, they nonetheless appear not to be detectable solely through the observed market variables used in the standard Merton model implementation. These implied probabilities can thus be used to adjust the standard implementation Merton computed spreads for names for which market spreads are not available. This approach is intuitively pleasing because historical volatilities would not incorporate jumps to ruin that in fact have not occurred for surviving names, although it is well known that there is always some positive probability of such events.

The factoring in of these implied probabilities of jump to ruin serves to bring the average loss given default from about the 20% predicted by the pure Merton approach for investment grade bonds to about 37%, which is much closer to statistically observed experience for senior bonds in the United States. However, this is still somewhat below the 45-50% range published by Moody's and factoring in the jump to ruin only serves to increase, albeit slightly, the already high probabilities of default implied by the pure Merton approach. This may suggest that in addition to the incorporation of implied probability of jump to ruin, the Merton model should also be modified to recognize default only if the firm value falls more than a certain percentage below the face value of the bonds. Such a change would simultaneously reduce the probability of default implied by the Merton model and increase the average loss upon default. The rationale for such a change is that firms often restructure and issue various forms of capital notes and preferred stock when faced with financial difficulties that may limit default situations to more significant failures.

In addition to the use of implied jumps to ruin and the provision of an allowance by which the firm's value may fall below the face value of debt without triggering a default, it may also be appropriate to incorporate the fact that historical volatility may not reflect current market conditions and that this might best be addressed by development of an adjustment factor to move the volatility factor from an historical basis to an implied and more current one. Implementation of this comprehensive approach would require determination of the "default allowance", testing, refinement and implementation of the method to compute implied volatilities, and computation of the implied jump to ruin. Some or all of these components may vary based on credit rating and other market factors. One key advantage of this approach is that implementation should be possible using only publicly available data. Another possible advantage and use of this approach might be that it could provide investment, actuarial, and regulatory constituencies with a tool to quantify expected default rates vis-à-vis historical default rates and to adjust capital requirements based solely on publicly available information depicting current market conditions.

*Pricing Accuracy*

The following table demonstrates how the models performed across study dates in aggregate for the 42 corporate issuers. The summary statistics for each study date are presented in Appendix B:

| | Regression Based Models | | | Merton Models | | | | |
|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | CreditGrades Standard |
| Average Deviation | | | | | | | | |
| Additive Basis | -22 | -5 | -43 | -49 | -25 | -33 | -48 | -20 |
| Percentage Basis | 41% | 100% | 21% | -53% | -3% | -35% | -41% | -6% |
| Abs Average Deviation | | | | | | | | |
| Additive Basis | 70 | 76 | 67 | 85 | 72 | 82 | 77 | 73 |
| Percentage Basis | 73% | 124% | 55% | 73% | 72% | 70% | 68% | 76% |

Table 3 – Results across all study dates

Based on the negative average deviation, all the models produced spreads that were, on average, clearly below the actual spreads. The Merton models produced, on average, the lowest spreads. However, L50, a Merton-type model, and the CreditGrades model, produced average deviations that appear to suggest that these models come closest to reproducing the overall market.

In terms of the accuracy for the individual names, the Credit Score, Distance to Default CG, L50, and CreditGrades produced the most accurate results, although the observed differences between models were small. Results for all other models were very similar with the standard Merton model producing the highest average absolute deviation.

Moreover, the pricing error of around 73% for the standard Merton model is consistent with other broader empirical research (see the study performed by Eom).

The following tables detail the results for the third most risky issuers as measured by the observed credit spreads and for the remaining less risky issuers.

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -23 | 3 | -14 | -19 | 1 |
| Percentage Basis | -53% | 5% | -33% | -43% | -3% |
| Abs Average Deviation | | | | | |
| Additive Basis | 31 | 31 | 29 | 30 | 34 |
| Percentage Basis | 70% | 77% | 68% | 71% | 82% |

Table 4 – Results for top two-thirds

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -102 | -80 | -72 | -105 | -61 |
| Percentage Basis | -52% | -19% | -37% | -38% | -11% |
| Abs Average Deviation | | | | | |
| Additive Basis | 198 | 153 | 189 | 169 | 151 |
| Percentage Basis | 79% | 61% | 74% | 62% | 63% |

Table 5 – Results for bottom one-third

For issuers with the lower two-thirds of credit spreads, both the CreditGrades and L50 models produced average spreads that are extremely close to the observed.

When considering only firms in the higher third of credit spreads, all the models including CreditGrades and L50 underestimated the market. Still the CreditGrades model has the best performance, coming in 61 basis points below the market and L50 came in 80 basis points below the market. Merton, on the other hand, was over 100 basis points below the market.

Based on the above results, we can draw the following conclusions:

Since the CreditGrades model outperforms the regression-based models in terms of average percentage deviation, but does not have any real performance advantage in average absolute deviations, the CreditGrades model does a better job of matching the overall market data, but does not necessarily offer a clear advantage on a firm-by-firm basis.

Although the CreditGrades model outperforms most of the Merton models, the results for the Merton model variant L50 are very close to those of the CreditGrades model. This suggests first that the CreditGrades and L50 models do a good job at matching the overall market, but less so at matching firm-by-firm data and second that L50, a Merton-like probabilistic model without some of the sophisticated machinery of the CreditGrades model such as variable default barriers, seems to have performance close to that of the CreditGrades model and perhaps even is correlated with the CreditGrades model in some way.

On average, the extra spread generated by assumption of a 50% loss upon default is sufficient to bring Merton modeled spreads in line with the observed investment grade spreads. Although this result held for average deviations, it did not carry through to the average absolute deviations suggesting that at the firm level, additional explanatory information is still needed.

Also, the use of a 50% loss upon default adds predictive power to the Merton model for higher spread firms and the additional machinery of the CreditGrades model yields yet more predictive power for these firms, but still not enough to bring average the CreditGrades spreads in line with the market.

These inferences led us to consider what could possibly be done to bring the CreditGrades spreads more in line with the market. One approach we took was to modify market spreads and assess what would happen if we did this modification so as to assure that the CreditGrades model had no average deviation from the market. As results presented below suggest this would not materially improve the predictive power of the CreditGrades model.

| | Regression Based Models | | | Merton Models | | | | |
|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | CreditGrades Standard |
| Average Deviation | | | | | | | | |
| Additive Basis | -3 | 15 | -23 | -29 | -5 | -13 | -28 | 0 |
| Percentage Basis | 43% | 102% | 23% | -48% | 7% | -29% | -37% | 3% |
| Abs Average Deviation | | | | | | | | |
| Additive Basis | 49 | 66 | 53 | 60 | 52 | 62 | 52 | 50 |
| Percentage Basis | 63% | 125% | 51% | 69% | 71% | 66% | 66% | 77% |

Table 6 – Results across all study dates after setting the average deviation for CreditGrades to 0

We then considered whether a similar analysis by credit ratings might help, but as the following table demonstrates, the average deviations and average absolute deviations by credit ratings did not appear to be sufficiently different than those in aggregate.

| | | Credit Rating | | |
|---|---|---|---|---|
| | All Firms | AA | A | BBB |
| Average Deviation | | | | |
| January 5th, 2001 | -37 | -53 | -18 | -37 |
| October 15th, 2002 | -38 | -55 | -10 | -19 |
| September 30th, 2003 | 30 | 37 | 23 | -3 |
| September 29th, 2004 | -10 | -11 | -6 | -13 |
| Average | -14 | -21 | -3 | -18 |
| Abs Average Deviation | | | | |
| January 5th, 2001 | 55 | 79 | 37 | 37 |
| October 15th, 2002 | 70 | 96 | 30 | 19 |
| September 30th, 2003 | 53 | 67 | 34 | 10 |
| September 29th, 2004 | 32 | 40 | 24 | 13 |
| Average | 53 | 71 | 31 | 20 |

Table 7 – CreditGrades deviations by Credit Ratings

We then decided to consider whether a linear regression of the market spreads on the CreditGrades spreads by credit ratings might provide some improvement. The following table details our results:

| | CreditGrades Models | |
|---|---|---|
| | Regression | Standard |
| Average Deviation | | |
| January 5th, 2001 | -3 | -37 |
| October 15th, 2002 | -1 | -38 |
| September 30th, 2003 | 1 | 30 |
| September 29th, 2004 | 6 | -10 |
| Average | 1 | -14 |
| Abs Average Deviation | | |
| January 5th, 2001 | 38 | 55 |
| October 15th, 2002 | 82 | 70 |
| September 30th, 2003 | 29 | 53 |
| September 29th, 2004 | 23 | 32 |
| Average | 43 | 53 |

Table 8 – Results for the regression modified CreditGrades model

The average absolute deviation of the refined model is 10 basis points less than the standard CreditGrades model. This suggests perhaps some modest improvement in the predictive ability at the firm level. At the global level, the average deviation was close to zero, suggesting that this approach helps to bring the CreditGrades predictions more in line with the market on average. This leads us to conclude that there might be a benefit to combining the probabilistic and statistical approaches for prediction of credit spreads.

## ***Analysis for each Study Date***[2]

*Study Date January 5th, 2001*

Corporates

The following table details results for the corporate issuers for the January 5th, 2001 study date:

| | Regression Based Models | | | Merton Models | | | | | CreditGrades Models | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | Hull | Standard | CG2 | CG3 |
| Average Deviation | | | | | | | | | | | |
| Additive Basis | -66 | -20 | -61 | -102 | -62 | -91 | -92 | N/A | -56 | -60 | -23 |
| Percentage Basis | -31% | 49% | -16% | -69% | -32% | -58% | -64% | N/A | -35% | -34% | 5% |
| Abs Average Deviation | | | | | | | | | | | |
| Additive Basis | 71 | 83 | 73 | 105 | 86 | 97 | 100 | N/A | 92 | 93 | 84 |
| Percentage Basis | 39% | 75% | 37% | 73% | 55% | 65% | 72% | N/A | 64% | 71% | 69% |

Table 9 – January 5th, 2001 Results (corporates)

Based on the negative average deviation, all the models produced spreads that were, on average, clearly below the actual spreads. The CreditGrades models undervalued spreads the least with the CG3 producing the lowest average percentage deviation. Meanwhile, the Merton models produced, on average, the lowest spreads. L50 was the only Merton-like model to produce the average deviation comparable to the CreditGrades and regression-based models, again showing the strength of this model with its one simple modification to the basic Merton approach.

In terms of the accuracy for the individual names, the regression-based models, in particular the Credit Score and the Distance to Default CG, produced the most accurate results, albeit by a small amount. Results for all other models were somewhat poorer with the results for the standard Merton model and Distance to Default KMV model producing the highest average absolute percentage deviation.

[2] The analysis presented in this section reflects results for **all** firms for which we had the necessary data.

Financials

The following table details results for the issuers in the financial services for the January 5th, 2001 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -35 | 21 | -21 | 14 | 51 |
| Percentage Basis | -52% | 39% | -28% | 11% | 66% |
| Abs Average Deviation | | | | | |
| Additive Basis | 37 | 49 | 31 | 51 | 81 |
| Percentage Basis | 56% | 69% | 46% | 74% | 109% |

Table 10 – January 5th, 2001 Results (financials)

From the above results, it is easy to observe that as the developers of the CreditGrades acknowledged, the CreditGrades model had the poorest performance. Meanwhile the Merton models performed similarly, with L50 coming in slightly on the high side and the standard implementation Merton model coming in slightly on the low side.

Top two-thirds

The following table details results for the issuers within the top two-thirds group for the January 5th, 2001 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -33 | 2 | -24 | -18 | 8 |
| Percentage Basis | -60% | -3% | -43% | -39% | 0% |
| Abs Average Deviation | | | | | |
| Additive Basis | 38 | 36 | 32 | 44 | 50 |
| Percentage Basis | 66% | 59% | 56% | 74% | 79% |

Table 11 – January 5th, 2001 Results (top two-thirds)

From the above results, it is easy to observe that for this study date, the CreditGrades and the L50 Merton model were, on average, very accurate for the less risky issuers. Meanwhile, other Merton models still produced spreads on average lower than the observed. However, in terms of individual accuracy the Merton models outperformed the CreditGrades model for these issuers.

Bottom one-third

The following table details results for the most risky one-third of issuers for the January 5th, 2001 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -194 | -135 | -178 | -169 | -113 |
| Percentage Basis | -76% | -44% | -67% | -63% | -37% |
| Abs Average Deviation | | | | | |
| Additive Basis | 195 | 162 | 183 | 180 | 168 |
| Percentage Basis | 76% | 57% | 69% | 69% | 64% |

Table 12 – January 5th, 2001 Results (bottom one-third)

Based on the above results, none of the models performed well for these type of issuers. However, the CreditGrades model did have the lowest average deviations followed by L50; while the L50 model had the lowest absolute average deviations, followed by CreditGrades. These results suggest that use of a 50% loss on a default adds significant predictive power to the standard Merton implementation and that CreditGrades' sophisticated machinery adds yet more predictive power, albeit that more remains to be done.

*Study Date October 15th, 2002*

Corporates

The following table details results for the corporate issuers for the October 15th, 2002 study date:

| | Regression Based Models | | | Merton Models | | | | | CreditGrades Models | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | Hull | Standard | CG2 | CG3 |
| Average Deviation | | | | | | | | | | | |
| Additive Basis | -96 | -65 | -90 | -104 | -64 | -85 | -105 | -105 | -52 | -61 | -12 |
| Percentage Basis | -18% | 35% | -16% | -55% | -18% | -41% | -55% | -55% | -17% | -28% | 15% |
| Abs Average Deviation | | | | | | | | | | | |
| Additive Basis | 109 | 113 | 103 | 121 | 99 | 109 | 115 | 114 | 98 | 95 | 87 |
| Percentage Basis | 46% | 90% | 46% | 75% | 63% | 69% | 67% | 69% | 68% | 67% | 73% |

Table 13 – October 15th, 2002 Results (corporates)

As with the January 5th 2001 results, all the models produced spreads that were, on average, clearly below the actual spreads. Once again, the CreditGrades models undervalued spreads the least with the CG3 producing the lowest average percentage deviation. Also, the Merton models produced, on average, the lowest spreads. L50 was again the only Merton-like model to produce the average deviation comparable to the CreditGrades models. Distance to Default KMV was the only regression-based model to produce average deviations close to the CreditGrades model.

In terms of the accuracy for the individual names, the Credit Score, Distance to Default CG, and L50 produced the most accurate results based on the average absolute percentage deviation. However, in term of the average absolute deviation the CreditGrades and L50 models slightly outperformed. The standard Merton model produced the highest average absolute deviation, and the Distance to Default KMV model produced the highest average absolute percentage deviation.

Financials

The following table details results for the issuers in the financial services for the October 15th, 2002 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -80 | 8 | -52 | -31 | 75 |
| Percentage Basis | -38% | 26% | -16% | -9% | 62% |
| Abs Average Deviation | | | | | |
| Additive Basis | 114 | 100 | 102 | 83 | 131 |
| Percentage Basis | 73% | 72% | 68% | 58% | 99% |

Table 14 – October 15th, 2002 Results (financials)

Again, the CreditGrades model had the poorest performance, while all Merton models outperformed the CreditGrades model based on the average and absolute average deviation bases with SVWavgV producing the most accurate results. It is interesting to notice that on average, CreditGrades produced spreads higher than the observed spreads.

Top two-thirds

The following table details results for the issuers within the top two-thirds group for the October 15th, 2002 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -30 | 5 | -17 | -27 | 8 |
| Percentage Basis | -46% | 2% | -28% | -43% | 3% |
| Abs Average Deviation | | | | | |
| Additive Basis | 54 | 49 | 51 | 45 | 56 |
| Percentage Basis | 77% | 72% | 72% | 67% | 82% |

Table 15 – October 15th, 2002 Results (top two-thirds)

The CreditGrades and the L50 Merton model were, on average, more accurate for the less risky issuers. Meanwhile, other Merton models still produced spreads on average lower than the observed. However, in terms of individual accuracy the Merton models outperformed the CreditGrades model for these issuers.

Bottom one-third

The following table details results for the most risky one-third of issuers for the October 15th, 2002 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -239 | -162 | -203 | -221 | -104 |
| Percentage Basis | -64% | -34% | -53% | -54% | -14% |
| Abs Average Deviation | | | | | |
| Additive Basis | 252 | 198 | 223 | 237 | 201 |
| Percentage Basis | 70% | 50% | 62% | 62% | 57% |

Table 16 – October 15th, 2002 Results (bottom one-third)

Again, neither model performed well. However, the CreditGrades model had the lowest average deviations; while the L50 model had the lowest absolute average deviations. The results shown again demonstrate that assumption of a 50% loss in the Merton model adds significant predictive ability and that the additional machinery of CreditGrades yields yet more power, albeit not enough to bring model spreads in line with observed spreads.

*Study Date September 30th, 2003*

Corporates

The following table details results for the corporate issuers for the September 30th, 2003 study date:

| | Regression Based Models | | | Merton Models | | | | | CreditGrades Models | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | Hull | Standard | CG2 | CG3 |
| Average Deviation | | | | | | | | | | | |
| Additive Basis | 20 | 38 | 9 | -3 | 29 | 13 | -6 | -5 | 36 | 19 | 58 |
| Percentage Basis | 74% | 187% | 67% | -15% | 57% | 14% | -15% | -14% | 56% | 24% | 103% |
| Abs Average Deviation | | | | | | | | | | | |
| Additive Basis | 36 | 62 | 36 | 52 | 52 | 55 | 40 | 48 | 59 | 52 | 73 |
| Percentage Basis | 82% | 202% | 82% | 82% | 102% | 87% | 71% | 74% | 111% | 94% | 141% |

Table 17 – September 30th, 2003 Results (corporates)

For this study date, only the standard, SVWavgV, and Hull Merton models produced on average spreads that were slightly lower than the observed and in fact these models essentially reproduced the observed spreads on average; all other models resulted in spreads that were on average higher than the actual spreads. CreditGrades Models, especially CG3, produced the largest spreads.

In terms of the accuracy for the individual names, the Credit Score and the Distance to Default CG produced the most accurate results based on the average absolute deviation. However, in terms of the average absolute percentage deviation, SVWavgV and Hull Merton had the lowest average absolute percentage deviation, albeit by small amounts.

Financials

The following table details results for the issuers in the financial services for the September 30th, 2003 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -3 | 66 | 22 | 42 | 128 |
| Percentage Basis | 20% | 175% | 71% | 117% | 270% |
| Abs Average Deviation | | | | | |
| Additive Basis | 43 | 74 | 50 | 64 | 134 |
| Percentage Basis | 81% | 188% | 103% | 148% | 284% |

Table 18 – September 30th, 2003 Results (financials)

As in prior study dates, the CreditGrades model had the poorest performance, while all Merton models outperformed the CreditGrades model based on the average and absolute average deviation bases with standard Merton Model and Jump producing the most accurate results.

Top two-thirds

The following table details results for the issuers within the top two-thirds group for the September 30th, 2003 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | 3 | 36 | 15 | 10 | 41 |
| Percentage Basis | -4% | 97% | 33% | 16% | 104% |
| Abs Average Deviation | | | | | |
| Additive Basis | 28 | 47 | 33 | 32 | 56 |
| Percentage Basis | 86% | 140% | 99% | 101% | 163% |

Table 19 – September 30th, 2003 Results (top two-thirds)

The standard Merton model produced, on average, credit spreads that were more accurate for the less risky issuers. The CreditGrades and the L50 Merton model had the worst performances both in terms of the average and absolute average deviations.

Bottom one-third

The following table details results for the most risky one-third of issuers for the September 30th, 2003 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -16 | 35 | 14 | -11 | 76 |
| Percentage Basis | -18% | 42% | 7% | -3% | 79% |
| Abs Average Deviation | | | | | |
| Additive Basis | 93 | 74 | 95 | 68 | 106 |
| Percentage Basis | 74% | 75% | 71% | 54% | 104% |

Table 20 – September 30th, 2003 Results (bottom one-third)

The Merton models, in particular SVWavgV and Jump, produced the best results for these issuers. The CreditGrades model significantly overestimated the probability of default for the most risky issuers on this study date.

*Study Date September 29th, 2004*

Corporates

The following table details results for the corporate issuers for the September 29th, 2004 study date:

| | Regression Based Models | | | Merton Models | | | | | CreditGrades Models | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | Hull | Standard | CG2 | CG3 |
| Average Deviation | | | | | | | | | | | |
| Additive Basis | 29 | 26 | -1 | -23 | -3 | -13 | -17 | -22 | 1 | -5 | 23 |
| Percentage Basis | 95% | 162% | 27% | -59% | -19% | -44% | -46% | -55% | -18% | -30% | 22% |
| Abs Average Deviation | | | | | | | | | | | |
| Additive Basis | 39 | 48 | 24 | 48 | 39 | 47 | 37 | 47 | 43 | 45 | 52 |
| Percentage Basis | 100% | 177% | 55% | 90% | 81% | 86% | 75% | 87% | 89% | 90% | 101% |

Table 21 – September 29th, 2004 Results (corporates)

For this study date, all Merton models produced on average spreads that were slightly lower than the observed; all other models resulted in spreads that were on average higher than the actual spreads. The standard CreditGrades model, the Distance to Default CG regression model, and L50 had average deviations of 1, -1, and -3 basis points, respectively, and L50 and the Standard CreditGrades Model had the best average percentage deviation at -19% and -18%, respectively. The Credit Score regression model resulted in the largest average deviations.

In terms of the accuracy for the individual names, the Distance to Default CG regression model produced the most accurate results based on the average absolute and average absolute percentage deviations. The CG3 model resulted in the highest average absolute deviation and the Distance to Default KMV regression model had the largest average absolute percentage.

Financials

The following table details results for the issuers in the financial services for the September 29th, 2004 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -23 | 17 | -10 | 30 | 68 |
| Percentage Basis | -32% | 69% | 3% | 85% | 162% |
| Abs Average Deviation | | | | | |
| Additive Basis | 35 | 40 | 29 | 51 | 88 |
| Percentage Basis | 96% | 122% | 88% | 138% | 215% |

Table 22 – September 29th, 2004 Results (financials)

As in prior study dates, the CreditGrades model had the poorest performance, while all Merton models outperformed the CreditGrades model based on the average and absolute average deviation bases with Jump model producing the most accurate results.

Top two-thirds

The following table details results for the issuers within the top two-thirds group for the September 29th, 2004 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -15 | 3 | -8 | -2 | 10 |
| Percentage Basis | -54% | 0% | -34% | -21% | 15% |
| Abs Average Deviation | | | | | |
| Additive Basis | 27 | 29 | 26 | 30 | 39 |
| Percentage Basis | 97% | 102% | 93% | 101% | 130% |

Table 23 – September 29th, 2004 Results (top two-thirds)

The L50 Merton model produced, on average, credit spreads that were more accurate for the less risky issuers as measured by the average percentage deviation. All other Merton models were undervaluing the credit spreads. For individual names, Merton model Jump produced slightly better results; while CreditGrades had the worst results.

<u>Bottom one-third</u>

The following table details results for the most risky one-third of issuers for the September 29th, 2004 study date:

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -39 | -4 | -21 | -19 | 21 |
| Percentage Basis | -55% | -9% | -38% | -25% | 13% |
| Abs Average Deviation | | | | | |
| Additive Basis | 83 | 59 | 79 | 58 | 75 |
| Percentage Basis | 78% | 62% | 72% | 56% | 76% |

Table 24 – September 29th, 2004 Results (bottom one-third)

On average, all Merton models had spreads lower than observed, although L50 was extremely close to observed spreads at an average deviation of -4 basis points; while the CreditGrades model produced spreads higher than observed. Individually, the Merton models SVWavgV followed closely by Jump produced the most accurate results.

Based on the results detailed above we can draw the following conclusions:

In all environments, the CreditGrades (both with the CreditGrades definition of debt and using book value of debt as the definition of debt) models had the highest average spreads followed closely by L50 model. This clearly helped the CreditGrades and L50 performance in higher spread environments, but was not as helpful in lower spread environments. In addition to reinforcing the suggestion of some type of correlation between the CreditGrades and L50 models, this suggests that different default points may be suitable depending upon the environment.

Of the option-theoretic models, the Merton model produced the lowest credit spreads, and the Jump, SVWavgV and Hull models produced credit spreads greater than Merton's but below those of the CreditGrades and L50 models.

It appears that changing the measurement of debt using the measures developed in this analysis would not materially improve the results as the standard CreditGrades and CG2 models produced nearly identical results in all credit spread environments. Relative to other CreditGrades models, the CG3 performed poorly in low credit spread environment, but its performance was similar to other models in high credit spread environment.

The Hull implementation of the Merton model performed similarly to the standard Merton model and SVWavgV model. This suggests that for the dates considered, the implied volatility skew obtained from the two-month options does not provide a pricing accuracy improvement upon the standard implementation of the Merton model.

For the individual issuers, some of the regression based models performed as well as some of the more probabilistic Merton and CreditGrades models. The fact that the credit score model performed as well as the other models suggests that in the long-term, the credit rating agencies are relatively accurate in determining the relative credit default risk for individual firms, if not in quantification of credit spreads.

All the models produced spreads that were higher on average relative to the observed for financials than for corporates except for the Merton Model in low spread environments. This suggests that models that may be suitable for one of corporates or financials may not be suitable for the other.

For corporates, CreditGrades and L50 appear to have very close results as do Merton and SVWavgV and Hull. Jump appears to be somewhere in the middle but weighted more heavily towards Merton and SVWavgV.

In all rate environments, the Distance to Default KMV regression had one of the worst performances. It significantly underperformed the regression model where CreditGrades definition of the distance to default was used. The following exhibit showing the movement of the average of two distance to default measures relative to the average spreads illustrates that while the distance to default measure defined in the CreditGrades model moved inversely to the credit spreads, the distance to default measure defined in the Moody's KMV model jumped up for the October 15th, 2002 study date when the spreads were the widest and then down for the September 30th, 2003 study date when the spreads narrowed. In our description of these models, we state that we expect a good distance to default measure to move inversely to the movement of the spreads. The Moody's KMV distance to default measure does not satisfy this expectation for the dates considered.

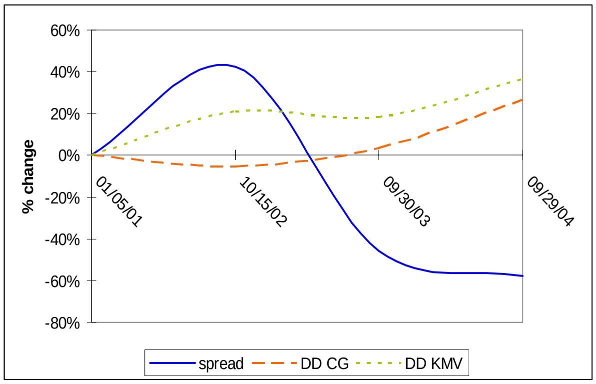


Exhibit 3 – Change in Average Spreads and Distance to Default Measures

In high spread environments, all the models produced spreads that on average were significantly (between 100bp and 240bp) below the observed for lower quality firms.

The CreditGrades and L50 models appear to have very close results for the less risky firms.

The random default barrier in the CreditGrades model appears to have the most significant effect for the most risky firms. In these situations, we find the widest divergence between the L50 and CreditGrades models, suggesting this effect of the random default barrier. In higher spread environments, this effect brings the CreditGrades' spreads closer to the observed, but, in lower spread environments, this effect moves the CreditGrades spreads farther away from the observed than are those of the L50 model. This suggests that in high spread environments, the markets may question firms' reported debt levels, while in low spread environments, the markets may be more prepared to accept firms' reported debt levels.

The results for Jump and SVWavgV may not have been as promising as originally anticipated. This could be because of the parameters selected, and further work could be done in this regard. Alternatively, for Jump, it could reflect a fundamental difference between the incorporation of jumps in an equity option model and the incorporation of jumps in a model in which we view a company's equity as a call option on its assets. In the former, significant jumps even if the equity does not approach ruin after the jump may play a significant role; while in the latter, it may be necessary to incorporate jumps to ruin (or at the least jumps to the default point) to assure that we are then able to reflect the resultant increase in defaults in the spread. This may also suggest that in fixed income, the role of jumps significantly outweighs the role of stochastic volatility, while in equities there tends to be somewhat more of a balance between the role that each plays, thereby perhaps explaining some of the results observed with SVWavgV.

**Observations**

Throughout our analysis, we observed that the L50 model results are often quite close to those of the CreditGrades model. Accordingly, we decided to examine just how close these two models were and analyzed first the average deviation and average absolute deviation between these two models and found that for corporates, the deviations were always small. We then did a linear regression of CreditGrades on L50 and found that on most of the dates studied, the CreditGrades spreads had a magnitude about 11% greater than those for L50 for corporates. The $R^2$ for this regression was greater than .9 on each of the four study dates.

To get a sense of the relationship between these two models independently of their relationship with the actual credit spreads, we looked at both the product moment partial correlation between CreditGrades and L50 and the Kendall Partial Rank-Order Correlation for CreditGrades and L50 and found that the product moment partial correlation was always greater than 90% for corporates and the Kendall Partial Rank-Order Correlation ranged from about 75% to 85% for Corporates, all of which suggests a relatively close relationship between these two models. Finally, we decided to examine which model produced a result closer to the actual spreads in more situations and found that on average, L50 produced a result closer to the actual spread than did CreditGrades in about 65% of the cases examined for Corporates. For financials, L50 came closer in about 75% of the cases examined.

When looking at all companies – both corporates and financials – but at the top two thirds of companies versus the bottom one third of companies, the relationships between CreditGrades and L50 are not quite as strong as those described above for all corporates regardless of size of spread. Further, for the bottom one third of companies, L50 performs better than CreditGrades only 49% of the time on the October 15th, 2002 study date. These results appear plausible because L50 has a deterministic default barrier, which may help it to do better for investment grade firms and poorer for lower quality firms than CreditGrades which has a stochastic default barrier.

The following table details the comparisons between L50 and CreditGrades models:

| | January 5th, 2001 | | October 15th, 2002 | |
|---|---|---|---|---|
| | Corporates | Financials | Corporates | Financials |
| $R^2$ for Linear Regression | 93% | 93% | 91% | 75% |
| Product Moment Partial Correlation | 94% | 94% | 90% | 87% |
| Kendall Partial Rank-Order Correlation | 84% | 84% | 82% | 84% |
| % of Names for which L50 is closer than CreditGrades | 68% | 75% | 62% | 73% |
| | September 30th, 2003 | | September 29th, 2004 | |
| | Corporates | Financials | Corporates | Financials |
| $R^2$ for Linear Regression | 92% | 61% | 94% | 59% |
| Product Moment Partial Correlation | 91% | 62% | 94% | 73% |
| Kendall Partial Rank-Order Correlation | 81% | 79% | 75% | 77% |
| % of Names for which L50 is closer than CreditGrades | 61% | 76% | 64% | 74% |

Table 25 – L50 and CreditGrades Models Comparisons (corporates vs. financials)

The following table details the comparisons between L50 and CreditGrades models for the third most risky issuers versus the remaining less risky issuers:

| | January 5th, 2001 | | October 15th, 2002 | |
|---|---|---|---|---|
| | Top ⅔ | Bottom ⅓ | Top ⅔ | Bottom ⅓ |
| $R^2$ for Linear Regression | 86% | 94% | 96% | 76% |
| Product Moment Partial Correlation | 90% | 95% | 98% | 82% |
| Kendall Partial Rank-Order Correlation | 85% | 79% | 84% | 69% |
| % of Names for which L50 is closer than CreditGrades | 77% | 54% | 71% | 49% |
| | September 30th, 2003 | | September 29th, 2004 | |
| | Top ⅔ | Bottom ⅓ | Top ⅔ | Bottom ⅓ |
| $R^2$ for Linear Regression | 89% | 76% | 74% | 80% |
| Product Moment Partial Correlation | 92% | 71% | 84% | 79% |
| Kendall Partial Rank-Order Correlation | 85% | 73% | 75% | 79% |
| % of Names for which L50 is closer than CreditGrades | 65% | 61% | 71% | 54% |

Table 26 – L50 and CreditGrades Models Comparisons (top two-thirds vs. bottom one-third)

In performing this comparative analysis, we set out to assess whether the probabilistic models such as CreditGrades provide any added value over regression-based models; whether the results achieved by the CreditGrades model could be achieved by a Merton-like probabilistic model; and the extent to which an enhanced model of this nature can be built that would have better performance than the CreditGrades model.

Based on our results, the random default barrier of the CreditGrades model may have a more significant effect for firms with higher credit spreads, and perhaps short term securities for which market data is not as readily available are also more significantly affected by the CreditGrades variable default barrier. These results are intuitively pleasing because firms with higher credit spreads often have high uncertainty associated with them even in the long-term, and in the very short term, there is also often significant uncertainty even about firms with low credit spreads.

Although the probabilistic models perform better in matching the credit spread movement than the regression-based models, we did not find them to have a significant advantage in pricing of the credit spreads for the individual issuers. However, some of the techniques presented in our analysis across study periods should be pursued for individual issuers as well.

A modified Merton model with loss on default always equal to 50% is less prone than the standard Merton model to undervalue credit spreads and appears to have performance consistent with that of the CreditGrades model, and perhaps even slightly better. This model produced credit spreads closer to the observed than those of the CreditGrades more than 60% of the time.

There is also suggestive evidence that in a dynamic marketplace, the predictive abilities of the CreditGrades model and other probabilistic models such as L50 may at times lag behind the market, suggesting the need for the development of tools to make better use of implied equity and asset volatilities in lieu of or in addition to historical equity volatility.

Finally, none of the models considered were very accurate in pricing credit spreads for the individual debt issuers. The following section details some of our suggestions to further the search for a model that will accurately describe the link between equity and debt prices, which, quoting Philipp Schönbucher, "*must* be there – somewhere".

**Suggestions for Further Analysis**

An alternative to the models considered is to implement a variant of both the Merton and CreditGrades models in which the market value of debt is based in part on the distance to default. In other words, rather than simply using the market value of debt that comes out of the Merton model or simply using the CreditGrades assumed market value of debt equal to one half the face value of debt, apply the following formula for the market value of debt, $D_t^*$, that reflects both the credit risk and the risk-free discounting:

$$D_t^* = D_t - (D_t - D_tL)\exp(-\gamma E_t/\sigma_E D_t L) \qquad 1.17$$

where $D_t = e^{-r(T-t)}D_T$, L is the expected loss upon default, and $\gamma$ is a factor to be applied to the distance to default measure to reflect the impact of distance to default on the market value of debt

In the CreditGrades model, we know that $\gamma = 0$ and L = .5.

We attempted to adjust both these factors in both the CreditGrades model and separately in the Merton model as a replacement for the Merton model's market value of debt to see if we could come up with a model with better results. Although our limited analysis did not succeed in this regard, it may be worthwhile to pursue additional optimization analysis to see what enhancements, if any, can be gleaned from this approach.

Another alternative is to notice that different levels of debt and default points may be suitable in different environments. Indeed, two of the basic differences among the models analyzed are the determination of the firm's total debt and the assumed default point. To identify how to determine a firm's total debt and how to select a suitable default point for the given environment, the following is an analytical method based solely on publicly available market data.

Merton found that the Black-Scholes formula continues to hold when we introduce a Poisson jump process with positive probability of immediate ruin as long as we replace the risk-free interest rate in the Black-Scholes formula with the sum of the risk-free rate and the Poisson parameter, s, which is equal to the expected number of jumps to immediate ruin per unit time. Extension of this insight to Merton's suggestion that we view a company's equity as a call option on its assets and recognizing the parameter s as the firm's spread will enable us to implicitly determine the level of debt and the default point assumed by the market.

Allen notes that the standard Merton model requires use of two simultaneous equations with two unknowns to solve for the value of the firm and for the standard deviation of changes in the firm value. After these values are computed, it is then straightforward to solve for probability of default, loss on default, market value of debt, and credit spread.

As an extension of this analysis, we could choose a universe of firms with publicly available spread data and view the firm's debt level as unknown and then use equations 1.2 and 1.6 along with the following variation of equation 1.5

$$s = -\frac{\log\{(A_t - E_t)/D_t\}}{(T-t)} \qquad 1.18$$

to solve for the three unknowns: (1) firm value, $A_t$ (2) standard deviation of changes in firm value, $\sigma_A$, and (3) debt level, $D_t$.

This analysis would then enable us to use standard statistical techniques to relate the $D_t$ thus computed with the debt levels computed by the methods described earlier and default points described earlier to determine what relationship if any exists in each type of market between the standard computed values and the implicitly computed values based on the extension of Merton. This could then form the basis for utilization of a standard type of Merton model with a redefined debt level based on the market situation for firms with unknown credit spreads.

**Implementation**

To test the probabilistic models designed and to produce the necessary output in real-time, a computer model in C/C++ was designed and implemented that runs through the input data for each selected time frame, computes results for each model under consideration and performs the required analytics.

Normally distributed random variables were obtained through implementation of a standard Box-Mueller routine, and the Cholesky decomposition was coded to support correlation between random variables for the stochastic volatility model.

Computation of the normal cumulative distribution function was performed both with the polynomial suggested in Hull for six decimal place accuracy and with the slower but more precise fifth order Gauss-Legendre quadrature for nine decimal place accuracy as suggested by Dridi.

For our purposes, the six decimal place accuracy was sufficient and therefore used for most of the testing.

To implement the standard Merton model, a two-dimensional Newton-Raphson routine was written, and to facilitate the implementation of the other models, the Merton model was also implemented via Monte-Carlo simulation.  To implement the Hull variant of the Merton model, a four-dimensional Newton-Raphson routine was written.

The statistical measures utilized, including the rank correlation statistics, were also implemented in the models.

The following tables detail the result statistics for the 42 corporate issuers for which we had all the necessary data for each study date. The aggregate results for the entire study period are presented in Tables 3 – 5 in the body of the report.

*Study Date January 5$^{th}$, 2001*

All corporates

| | Regression Based Models | | | Merton Models | | | | |
|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | CreditGrades Standard |
| Average Deviation | | | | | | | | |
| Additive Basis | -82 | -35 | -77 | -113 | -77 | -103 | -105 | -75 |
| Percentage Basis | -28% | 49% | -19% | -70% | -35% | -59% | -65% | -39% |
| Abs Average Deviation | | | | | | | | |
| Additive Basis | 89 | 99 | 84 | 115 | 93 | 107 | 110 | 93 |
| Percentage Basis | 37% | 75% | 33% | 71% | 53% | 63% | 72% | 60% |

Table B1 – January 5$^{th}$, 2001 Results (42 corporate issuers)

Top two-thirds

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -43 | -16 | -35 | -39 | -20 |
| Percentage Basis | -68% | -27% | -55% | -64% | -35% |
| Abs Average Deviation | | | | | |
| Additive Basis | 44 | 33 | 38 | 47 | 39 |
| Percentage Basis | 69% | 51% | 59% | 75% | 63% |

Table B2 – January 5$^{th}$, 2001 Results (top two-thirds)

Bottom two-third

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -254 | -198 | -238 | -236 | -184 |
| Percentage Basis | -75% | -49% | -67% | -67% | -47% |
| Abs Average Deviation | | | | | |
| Additive Basis | 256 | 213 | 246 | 236 | 200 |
| Percentage Basis | 76% | 56% | 71% | 67% | 56% |

Table B3 – January 5$^{th}$, 2001 Results (bottom one-third)

*Study Date October 15th, 2002*

All corporates

| | Regression Based Models | | | Merton Models | | | | |
|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | CreditGrades Standard |
| Average Deviation | | | | | | | | |
| Additive Basis | -76 | -52 | -95 | -92 | -61 | -72 | -94 | -55 |
| Percentage Basis | 1% | 46% | -3% | -56% | -12% | -40% | -51% | -14% |
| Abs Average Deviation | | | | | | | | |
| Additive Basis | 100 | 104 | 113 | 99 | 92 | 84 | 101 | 86 |
| Percentage Basis | 50% | 90% | 44% | 65% | 59% | 59% | 61% | 60% |

Table B4 – October 15th, 2002 Results (42 corporate issuers)

Top two-thirds

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -24 | 9 | -13 | -21 | 8 |
| Percentage Basis | -49% | 7% | -29% | -43% | -2% |
| Abs Average Deviation | | | | | |
| Additive Basis | 35 | 37 | 31 | 32 | 38 |
| Percentage Basis | 62% | 64% | 56% | 59% | 67% |

Table B5 – October 15th, 2002 Results (top two-thirds)

Bottom two-third

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -223 | -201 | -189 | -239 | -181 |
| Percentage Basis | -71% | -49% | -62% | -66% | -45% |
| Abs Average Deviation | | | | | |
| Additive Basis | 227 | 201 | 189 | 239 | 181 |
| Percentage Basis | 71% | 49% | 62% | 66% | 45% |

Table B6 – October 15th, 2002 Results (bottom one-third)

*Study Date September 30th, 2003*

<u>All corporates</u>

| | Regression Based Models | | | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | | | | |
| Additive Basis | 27 | 40 | 3 | 13 | 35 | 34 | 12 | 42 |
| Percentage Basis | 92% | 174% | 82% | 20% | 64% | 11% | -3% | 62% |
| Abs Average Deviation | | | | | | | | |
| Additive Basis | 42 | 58 | 46 | 64 | 58 | 73 | 47 | 65 |
| Percentage Basis | 101% | 189% | 96% | 70% | 102% | 77% | 63% | 106% |

Table B7 – September 30th, 2003 Results (42 corporate issuers)

<u>Top two-thirds</u>

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | - 3 | 27 | 8 | 1 | 25 |
| Percentage Basis | -19% | 81% | 15% | -7% | 68% |
| Abs Average Deviation | | | | | |
| Additive Basis | 19 | 34 | 22 | 19 | 35 |
| Percentage Basis | 65% | 116% | 76% | 70% | 118% |

Table B8 – September 30th, 2003 Results (top two-thirds)

<u>Bottom two-third</u>

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | 45 | 49 | 86 | 35 | 76 |
| Percentage Basis | -20% | 30% | 4% | 3% | 51% |
| Abs Average Deviation | | | | | |
| Additive Basis | 154 | 106 | 173 | 102 | 124 |
| Percentage Basis | 80% | 73% | 79% | 51% | 82% |

Table B9 – September 30th, 2003 Results (bottom one-third)

*Study Date September 29th, 2004*

## All corporates

| | Regression Based Models | | | Merton Models | | | | |
|---|---|---|---|---|---|---|---|---|
| | Credit Score | DD KMV | DD CG | Standard | L50 | Jump | SVWavgV | CreditGrades Standard |
| Average Deviation | | | | | | | | |
| Additive Basis | 42 | 29 | -3 | -4 | 4 | 8 | -4 | 8 |
| Percentage Basis | 97% | 130% | 25% | -64% | -30% | -50% | -45% | -31% |
| Abs Average Deviation | | | | | | | | |
| Additive Basis | 50 | 43 | 24 | 62 | 45 | 64 | 48 | 49 |
| Percentage Basis | 101% | 142% | 48% | 86% | 72% | 81% | 76% | 78% |

Table B10 – September 29th, 2004 Results (42 corporate issuers)

## Top two-thirds

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | -20 | -9 | -16 | -16 | -11 |
| Percentage Basis | -76% | -40% | -63% | -58% | -47% |
| Abs Average Deviation | | | | | |
| Additive Basis | 24 | 22 | 23 | 22 | 23 |
| Percentage Basis | 85% | 75% | 79% | 81% | 82% |

Table B11 – September 29th, 2004 Results (top two-thirds)

## Bottom two-third

| | Merton Models | | | | CreditGrades |
|---|---|---|---|---|---|
| | Standard | L50 | Jump | SVWavgV | Standard |
| Average Deviation | | | | | |
| Additive Basis | 27 | 30 | 55 | 21 | 44 |
| Percentage Basis | -41% | -9% | -23% | -21% | 0% |
| Abs Average Deviation | | | | | |
| Additive Basis | 136 | 92 | 148 | 101 | 100 |
| Percentage Basis | 87% | 66% | 84% | 66% | 69% |

Table B12 – September 29th, 2004 Results (bottom one-third)

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