Review of "Distortion Risk Measures and Economic Capital", Werner Hurlimann, North American Actuarial Journal, Volume 8, Number 1, January, 2004, pages 86-95.

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This is a very interesting paper, reflecting solid mathematics, careful research, and a thorough review of the literature.

The reader needs to review and reflect on coherent risk measures, distortion functions, dual distortion functions, distorted distribution functions, dual distorted distribution functions, distortion (risk) measures, dual distortion (risk) measures, survival functions, distorted survival functions, dual distorted survival functions, and stop-loss transforms. This reviewer found it useful to trace through many of these concepts with four distortion functions.

They were: 1) $g(x) = x$, $0 \le x \le 1$; 2) $g(x) = x^2$, $0 \le x \le 1$;
3) $g(x) = \sqrt{x}$, $0 \le x \le 1$; 4) $g(x) = x^p$, $0 \le x \le 1$, $0 < p < 1$.

In addition to the excellent References contained in the article, the reader may wish to review Value At Risk as explained in Financial Economics, Harry H. Panjer, Editor, The Actuarial Foundation, Schaumburg, Illinois, 1998.

The paper contains five examples concerning active risk management, the determination of capital requirements of an insurance business, and other topics.

Section 7 of the paper is devoted to the topic "Application to Economic Capital". To record some of its notation,

X = the insurance loss (e.g., claims minus premiums) at the end of a one-year period;

r = risk-free interest rate;

i = rate at which an insurer borrows some economic capital $C = EC[X]$;
g(t) = distortion function;

$g^{-1}(y)$ = inverse function of g(t).

A formula is obtained for the end of the period value of the optimal economic capital.

This says that

$$(1 + r)\ EC\ [X] = F_X^{-1}\left(1 - g^{-1}\left(\frac{i-r}{1+r}\right)\right)$$

where $F_X(x)$ is the distribution function of losses, and $F_X^{-1}(y)$ is its inverse function.

This formula identifies (1 + r) EC [X] with the Value At Risk of X at the confidence level

$$\alpha = 1 - g^{-1}\left(\frac{i-r}{1+r}\right).$$

There is a similar formula for the end of the period value of the minimum cost of capital.

Under suitable assumptions on insurance market prices, the collection of possible losses, and the distortion function, Theorem 6.3 of the paper implies the unique choice $g(t) = \sqrt{t}$. Hence the optimal confidence level for capital requirement should be

$$\alpha = 1 - \left(\frac{i-r}{1+r}\right)^2 .$$

The paper concludes with an example in which

$$F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

where $\Phi(x)$ is the standard normal distribution, μ is the mean, and σ is the standard deviation. For the choices i = 7.5%, r = 3.75%, and $g(t) = \sqrt{t}$, one obtains the optimal economic capital formula

$$EC[X] = \frac{1}{1+r}(\mu + 3.01\sigma).$$

It could be noted that with $g(t) = \sqrt{t}$, $g^{-1}(y) = y^2$, $\alpha = 1 – 0.0013$, and $F_X^{-1}(1 – 0.0013)$ implies that $\frac{x-\mu}{\sigma} = 3.01$ or $x = \mu + 3.01\sigma$.