

Review of
Extremes and Integrated Risk Management
Paul Embrechts, Editor
Risk Waters Group, 2000.

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Introduction

Extremes and Integrated Risk Management is an edited volume, compiling nineteen papers by various authors on the topic of extreme value theory (EVT) and its application. The book is divided into four sections: Basic Extreme Value Theory, Risk Measures and Extreme Value Theory, Applications to Finance, and Applications to Insurance. This note briefly reviews each section in turn, and finishes with some concluding remarks.

Prelude

Before diving into a description of the volume proper, it should be noted that the introductory material includes a paper entitled "The Bell Curve is Wrong: So What?" by Embrechts. This brief article gives a non-technical argument that the ubiquitous Gaussian normal distribution is often applied in situations that call for the use of other probability distributions. Those other distributions are a central theme of EVT -- and of *Extremes and Integrated Risk Management* -- so this article motivates the remainder of the volume well. If the reader has already accepted the notion that many of the distributions encountered in their work have fatter tails than those of a Gaussian distribution -- and most probably have -- this introductory article will simply reinforce that belief.

Section I: Basic Extreme Value Theory

Three of the most important ideas underlying EVT are that:

1. The tails of many distributions can be approximated in a sense that can be made precise by the Generalized Pareto (GP) distribution, whose distribution function is given by

$$G(x; \mathbf{x}, \mathbf{b}) = 1 - \left(1 + \frac{x\mathbf{x}}{\mathbf{b}} \right)^{\frac{1}{\mathbf{x}}} \text{ for nonzero values of } \mathbf{x} .$$
 Consequently, estimating the value of \mathbf{x}

for the GP distribution which best matches the tails of an empirical data set goes a long way toward quantifying the likelihood of obtaining specified extreme results among future samples.

2. For a wide array of distributions as the number of observations in a random sample from that distribution increases, the distribution of the maximum of those observations in some sense comes to resemble the Generalized Extreme Value (GEV) distribution, whose distribution function is given by

$$H(x; \mathbf{x}) = \exp \left\{ - \left(1 + x\mathbf{x} \right)^{\frac{1}{\mathbf{x}}} \right\} \text{ again for nonzero values of } \mathbf{x} .$$
 Just as the Central Limit Theorem

shows that Gaussian distributions are central to the analysis of means of random samples, this result demonstrates that GEV distributions are crucial to an understanding of the maximum of such samples.

3. Situations involving multiple random variables cannot be analyzed with only a knowledge of those variables' marginal distributions. Especially in the prediction of extreme results, a full understanding

of the variables' joint dependence is crucial. "Copulas" are functions which can be used to study and specify this dependence.

The first paper in this section, "Extreme Value Theory for Risk Managers" by McNeil, provides an excellent introduction to these three fundamental ideas. The notation for the distribution functions given above largely follow that used in McNeil's paper. This overview of EVT should be required reading for any actuary who works with the tails of probability distributions, which is likely just about every actuary. The paper emphasizes univariate EVT, where the ideas in (1) and (2) above play an important role.

"Measuring Risk with Extreme Value Theory" by Smith gives a very brief overview of some of the classic EVT ideas mentioned above and goes on to describe several topics related to more current research. The material here is probably less relevant and accessible to most actuarial audiences.

An important problem in applying the ideas discussed in bullet point (1) above is setting the cutoff between the bulk of an observed distribution and its tail. The GP distribution is only assured to match the underlying distribution far out in the tail, but limiting the "tail" to only very extreme events will leave little data for the estimation of the parameter α of the GP distribution. "Adaptive Threshold Selection in Tail Index Estimation" by Matthys and Beirlant, the third paper in this section, discusses various approaches to this dilemma. This chapter is more technical than most of the remainder of the book.

The fourth chapter in this section, "Pitfalls and Opportunities in the Use of Extreme Value Theory in Risk Management" by Diebold et al, is an expository piece which argues that EVT should play a role in financial risk management, but that its role should be somewhat limited. The authors claim, for example, that EVT techniques are not really necessary for the analysis of events representing a 5th or a 1st percentile result, but more applicable to estimation of a 0.1th percentile. While it is true that the fundamental EVT theorems are asymptotic in nature -- so roughly the farther the analysis is out in the tail, the better the results -- other EVT practitioners would almost certainly argue that their techniques shed light on more routine (such as 5th percentile) events. The article is a quick, non-technical read, and the view expressed by the authors is worthy of consideration.

"Modelling Multivariate Extremes" by Embrechts et al, discusses copulas and other related functions which help to quantify multiple variables' dependence structure. This paper is a well written and informative introduction to a number of ideas from multivariate EVT, and falls on the required reading list along with McNeil's paper.

Section II: Risk Measures and Extreme Value Theory

This short section includes just two papers, both of which discuss the advantages and disadvantages of various metrics used to analyze stochastic behavior.

"Correlation: Pitfalls and Alternatives" by Embrechts et al, gives an excellent critique of the measure most commonly used to quantify the interaction of random variables: correlation. There are serious flaws in the correlation statistic which, despite being elementary in nature, do not seem to be well known. The authors argue for the use of rank correlation, or preferably copulas, in the analysis of multiple random variables. This article is brief and non-technical so it takes little time to read, but the information is important and transcends the topic of EVT; it is the final entry on the required reading list.

VAR is currently a widely used risk measure. The second article in this section, "Thinking Coherently" by Artzner et al, points out that VAR can exhibit non-intuitive behavior. For example, if P and Q are two portfolios, it is possible for $\text{VAR}(P+Q)$ to be larger than $\text{VAR}(P) + \text{VAR}(Q)$. That is, VAR is not sub-additive. "Coherent" risk measures are sub-additive in addition to satisfying some other reasonable properties. This paper defines coherence and characterizes the class of coherent risk measures as precisely the measures that arise from a certain "generalized scenario" computation. This last topic may not be of

interest to the actuary, but the early part of the paper -- especially the critique of VAR -- provides practical and interesting information.

Section III: Applications to Finance

This section is by far the longest in the volume; it contains eight papers applying the ideas introduced in Section I to observations and analysis of financial returns.

"Value-at-Risk and Extreme Returns" by Danielsson and de Vries discusses the computation of a portfolio's VAR using EVT techniques and compares the results of these calculations to those derived by a currently more widespread methodology: that embedded in the *RiskMetrics* software. "From Value-at-Risk to Stress-Testing: The Extreme Value Approach" by Longin considers similar questions, but comes at the analysis from a different and more theoretical approach.

An explanation of the methodology used on a world-wide-web site to apply EVT calculations in real time to financial market returns and backtests of the subsequent predictions appears in "Reading the Riskometer" by McNeil. The website was disabled in November 2000, but this article may still be useful in convincing the reader that EVT computations do offer more predictive reliability than methods based upon an assumption of normal returns.

"Extreme Value Theory: An Empirical Analysis of Equity Risk" by Gavin analyzes return data on a number of equity markets from around the globe. This paper includes pragmatic comments on a number of issues that arise in attempts to apply EVT techniques to real-world data.

Mikosch and Starica contributed "Is it really Long Memory we See in Financial Returns?". This article includes a time series analysis of S&P returns, looking at autocorrelation associated with very long lags. The article is quite technical and uses techniques that do not seem to have any relation to the theory described elsewhere in the volume. Consequently, it does not seem to fit in this collection.

"Multivariate Extremes for Foreign Exchange Data" by Starica is another very technical paper demanding a good deal of mathematical sophistication of the reader. It details methods of analyzing the dependence structure of the stochastic movement of multiple currencies. Even if the reader does not want to wade through the mathematical details, the data presented in Figure 1 is striking and informative. It gives remarkably strong support to the adage that "in a crisis, correlations go to 1". Analysis of this phenomenon without the striking graphic but with more accessible analysis appears in Straetmans' "Extremal Spill-Overs in Equity Markets".

The final chapter in this section, "Modelling and Measuring Operational Risk" by Cruz et al, uses GP distributions to model operational risk. This is a great example of an application of EVT to the quantification of a risk often thought to be intractable to rigorous quantitative techniques.

Section IV: Applications to Insurance

This section contains three chapters. The first, "Extreme Value Statistics and Wind Storm Losses: A Case Study" by Rootzén and Tajvidi, contains some information that will only be useful to those insuring losses due to wind storms. The majority of the text, however, is dedicated to a review of EVT theory and an explanation of how to apply that theory to a practical problem. This discussion is clearly written, of practical use, and is well worth the reader's time.

"Bayesian Risk Analysis" by Smith and Goodman also applies EVT techniques to losses that might be encountered by a P&C insurer. While the analysis is not recounted in fine technical detail, a wide range of techniques are applied here demanding a solid statistical background on the part of the reader.

The concluding paper in this volume, "Developing Scenarios for Future Extreme Losses Using the Peaks-over-Threshold Method" by McNeil and Saladin, applies EVT in a frequency/severity framework to losses due to natural catastrophes. A good deal of mathematical detail is included and consequently this is not an easy chapter to read. It might provide good background, however, for someone faced with their own frequency/severity data to analyze.

Summary Remarks

Extremes and Integrated Risk Management has three primary strengths for consumption by life actuaries looking for an introduction to EVT. First, it gives a not-too-technical overview of the fundamental theory behind EVT and inspiration to the reader to apply that body of theory to a broad range of situations. Second, as an edited volume, the audience gets to see the theory described and applied by authors with a number of different perspectives. Third, it presents applications to a number of different financial models or data sets.

The book has a some drawbacks, too. The first to present itself to the reader is the pricetag: at the time of this writing the book is being offered on Amazon.com for \$230, almost a dollar per page. Furthermore the volume is not entirely self-contained in the sense that, while the theoretical material is communicated clearly by a number of authors, it is covered repeatedly at a relatively low level of sophistication which is inadequate to follow all the intricacies of the analysis in many of the applications. Finally, none of the examples is directly applicable to life insurers' business.

In conclusion, this book might best be viewed as a complement to a textbook on EVT. This volume provides a good view of the primary EVT theorems and conveys a number of examples putting these results into action, while the textbook could provide a more rigorous and thorough grounding in the subject.