The “Greeks”

The “Greeks" are five partial derivatives of price with respect to the parameters or factors which determine the value of an option. They can be used as indicators to help monitor and analyze the risks associated with portfolios which include options. The Greeks and the associated derivatives are:

- Delta (price of underlying)
- Gamma (2^{nd} derivative of price)
- Vega (volatility)
- Theta (time)
- Rho (risk free interest rate)

Delta:

The delta of an option is the option’s sensitivity to market movement (the units of change in the price of the option, per unit of change in the price of the underlying market instrument). In other terms, delta is also the slope of the price curve of the option to the market price of the underlying instrument. Expressed mathematically, delta is the partial derivative of the option price with respect to the underlying market instrument.

$$\Delta = \frac{\partial option}{\partial asset}$$

Example:

Using the Black-Scholes European call option model delta is equal to $N(d_1)$ or N((ln(Price/K) +(r+Variance/2)*(time to expiration))/squareroot(variance*time to expiration)), where K is the exercise price, r is the risk free rate for the time to expiration, and N is the standard normal cumulative distribution function.

An at the money call has a delta that tends to 0.5 (from above) as the time to expiration tends to zero. A delta of 0.5 means that a $1.00 increase in the price of the underlying asset will increase the price of the option by approximately $0.50. A $1.00 decrease in the price of the asset will decrease the price of the option by approximately $0.50.

Call options have deltas between 0 (0 probability of having value) and 1(long underlying asset). Put options have deltas between –1 and 0.In-the-money options have deltas that approach 1 for calls and –1 for puts. Out-of-the-money options have deltas that approach zero.

Common Uses:

Delta is the primary indicator used when monitoring option risk. Most often, delta is used as the "hedge ratio". By taking an opposite position in the underlying instrument equal in size to the option’s delta, we immunize the position against profit or loss variability due to small movements in the market. This is often referred to as delta hedging, or creating a delta-neutral portfolio. Delta hedging is the same process as hedging through duration matching in a fixed income portfolio. A risk management report could summarize the total equity exposure by summing up the products of amounts exposed times the delta’s for each equity, equity index, and any options in the portfolio.

Gamma:

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The gamma of an option is the rate of change of the option's delta with respect to the price of the underlying asset. It is the second partial derivative of the option value with respect to the asset price.

$$\Gamma = \frac{\partial \Delta}{\partial asset} = \frac{\partial^2 option}{\partial asset^2}$$

Example:

If gamma has a value of 0.2, this means that a $1.00 increase in the price of the underlying asset will increase the delta of the option by approximately 0.2. A $1.00 decrease in the price of the asset will decrease the delta of the option by approximately 0.2.

When gamma is small in absolute terms, delta changes slowly with changes in the underlying asset price. However, if gamma is large in absolute terms, delta is highly sensitive to the changes in the underlying asset price.

At-the-money options generally have the largest gamma. The further an option goes in- or out-of-the-money, the smaller the gamma generally becomes. Gamma is always positive for a call option.

Common Uses:

Delta plays the same role in approximating the sensitivity of an option's price to changes in the price of the underlying asset as duration does for measuring the sensitivity of a bond's price. In both cases, the changes are approximations which are only accurate for small changes in market prices. In both cases, the approximation can be improved through the use of second derivatives. For bonds, the second derivative is called convexity. For an option, the second derivative is often referred to as gamma.

If gamma is large, creating a delta-neutral portfolio may not provide adequate immunization against asset price changes. Delta hedging can be repeated more frequently or the option position can be made "delta- and gamma-neutral". This is done by taking a position in the underlying asset and an option on the asset such that the delta and gamma of this portfolio is equal and opposite in sign to the option being hedged. Again, this is essentially the same process as matching duration and convexity to obtain an immunized fixed-income portfolio.

Vega (also referred to as kappa or lambda):

The vega of an option is the rate of change of the value of the option with respect to the volatility of the underlying asset. Actual volatility over a period of time can be calculated. This will likely differ from the implied volatility calculated by fitting actual market prices to the volatility parameter in the various option models. Market volatility is not constant, although some option valuation formulas assume that it is.

$$V = \frac{\partial option}{\partial s}$$

s refers to the expected volatility of the underlying instrument over maturity period of the option.

Example:

If vega has a value of 0.5, this means that a 1 percent absolute increase (e.g., from 14% to 15%) in the volatility of the underlying asset will increase the price of the option by approximately $0.50.

A 1 percent decrease in the volatility of the asset will decrease the price of the option by approximately $0.50.

The vega of a long option position is always positive. At-the-money options generally have the largest vegas. Vega generally decreases as an option goes further in- or out-of-the-money. The vega of a forward contract, or underlying asset is zero.

Common Uses:

Vega calculated from historical prices, does not usually result in option prices consistent with market prices when using a lognormal pricing model which assumes constant volatility (Black-Scholes, e.g.), The reasons for this are that the market does reflect, to some extent, the nonnormal distribution of returns, and the expectation for future volatility that is different from historical volatility. An implied volatility or a matrix of implied volatilities is often calculated from the other variables used in the pricing formula (option price, asset price, time to maturity, risk-free rate, and exercise price).

Vega is not always appropriate for comparing the effect of a change in volatility on the price of different options because it measures absolute changes in volatility rather than relative changes. If the change in the option price due to a change in relative volatility is more important, normalized vega should be used. "Normalized vega" measures the percentage change in an option value for a 1 percent relative increase (e.g. from 14% to 14.14%) in implied volatility. Normalized vegas for in- and out-of-the-money options are often substantially larger than for at-the-money options.

Theta:

Theta is an indicator of an option's sensitivity to time. The theta of an option measures the units of change in the option price for a 1-day decrease in days remaining to expiration. Theta is often referred to as the time decay of an option.

$$\Theta = \frac{\partial option}{\partial t} \quad \text{where t is time}$$

Example:

If theta has a value of -$0.02, this means that a 1 day decrease in the time to expiration will decrease the price of the option by approximately $0.02. Assuming all other factors constant, the longer the time to option expiration, the larger the option price. Therefore, theta is generally a negative value because option price generally decreases as the time to expiration approaches.

Theta is generally the largest (in absolute value) for at-the-money options. In-the-money options typically have a larger theta than out-of-the-money options.

Common Uses:

Theta is generally not directly used to hedge option positions. Since there is no uncertainty as to the passage of time, one does not try to hedge its effect. However, it is useful as an aid in figuring out how the value of an option "depreciates" as time passes, and in planning for future transactions and transaction costs to keep delta in balance. In other words, since the option's value changes even when the underlying asset price remains the same, it is possible to separate the effect of time on the value of the option. Theta is related to gamma in a delta-neutral portfolio, normally with opposite sign.

Rho:

The rho of an option measures the change in an option price with respect to the domestic risk-free interest rate.

$$rho = \frac{\partial option}{\partial r}$$ where r is the risk-free interest rate

Example:

If rho is equal to 0.5, this means that a 1 percent absolute increase (e.g., from 4% to 5%) in the risk-free interest rate will increase the price of the option by approximately $0.50. A 1 percent decrease in the risk-free interest rate will decrease the price of the option by approximately $0.50.

Rho is always positive for European calls and always negative for European puts. Therefore, as interest rates increase, call option values will rise and put option values will fall.

Common Uses

Similar to theta, rho is not commonly used as a hedge parameter. However, it is a valuable statistic because it shows how sensitive an option is to changes in interest rates. It can be quite critical in pricing products which contain options (e.g. Equity Indexed Annuities), to understand how product margins need to change as interest rates move up and down.

The derivative of price with respect to the strike price (Eta) has not been mentioned above. Also, the derivative of price with respect to carry costs is sometimes referred to as “rho-b”, Many of these derivatives have closed form formulas, depending on the underlying distribution of market returns assumed in the option valuation formula.

Strengths and Weaknesses of the Greeks

The concept of using partial derivatives in pricing and hedging options and other financial instruments is well known and mathematically correct. It is necessary to evaluate some of these derivatives to understand the risk in a portfolio. However, the partial derivatives of option prices depend on the underlying distribution of price movements, and on price volatility of the underlying financial instruments. The weakness comes in using statistical models to calculate these values, which make assumptions that have been disproved from historical data. For example, it has been shown that stock market returns do not have a lognormal distribution; returns have much fatter tails, and return volatility varies by the length of the period measured. These models (lognormal) have worked reasonably well for some purposes, especially with adjustments to improve the fit to market prices. The practitioner needs to keep in mind the implicit assumptions when he draws inference from these calculations. It is always appropriate to compare derivatives (delta) from actual market prices to those obtained from the statistical models.

Implementation Issues

As indicated above, certain assumptions about price distributions (lognormal) may lead to closed form solutions for the partial derivatives. Software which calculates the Greeks for lognormal return distributions is readily available (Hull reference below or internet), or can be written fairly quickly in a spreadsheet, or a programming language. The lognormal type formulas require the generation of values of the normal distribution, and there are many sources for mathematical algorithms to do this.

Other considerations would include the size and number of option positions in the portfolio, the frequency of data needed for monitoring the portfolio. As the size and frequency become larger, there is more need for automatic processes to feed the data into the monitoring system. This may justify the acquisition of a commercial system which incorporates the acquisition of market data with the determination of modeled prices and partial derivative calculation.

Bibliography

Boyle, Phelim P.(1992). *Options and the Management of Financial Risk*. Society of Actuaries, Chapter 7.

Chriss, Neil A.. Black-Scholes and Beyond (ppg 162-181).

Hull, John C. *Options, Futures, and Other Derivatives (4th edition),* Prentice-Hall 2000, Chapters 11-13, especially pp. 307-335.

Sooran, Chand. *An Introduction to Hedging the Greeks.* Victory Risk Management Consulting, Inc. (http://www.finpipe.com/introhedge.htm)

Option Guide: The Greeks – Mr. Stock Inc. (http://www.mrstock.com/resourceCenter/greeks.html)

Pelsser, A., and T. Vorst. "The Binomial Model and the Greeks." *Journal of Derivatives*, Spring 1994, pp. 45-49.

Peters, Edgar E. *Chaos and Order in the Capital Markets.* John Wiley & Sons (1991).

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